VECTORS

Scalar

↓

Magnitude

Vector

↙ ↘

Magnitude Direction

Scalar quantity

⇒ Work

⇒ Speed

⇒ Distance

Vector quantity

⇒ Force

⇒ Velocity

⇒ Displacement

Representation of Vector

length = Magnitude

$|A| = 4$ unit (Magnitude)

$\vec{A} = 4$ unit south (Direction)

Types of Vector

1. Equal and unequal.
2. Parallel and antiparallel.
3. Collinear.
4. Concurrent.
5. Coplanar.
6. Zero
7. Unit.

⇒ Equal and unequal Vector

For two vectors to be similar $\vec{A}$ and $\vec{B}$ should have equal magnitude and have same direction.

$$\underbrace{\vec{V_1} = 5 m/sec \text{ East} \qquad \vec{V_2} = 5 m/sec \text{ West}}_{\text{Unequal vectors.}}$$

⇒ Parallel Vector

* Same direction
* $\theta = 0^\circ$

⇒ Antiparallel Vector

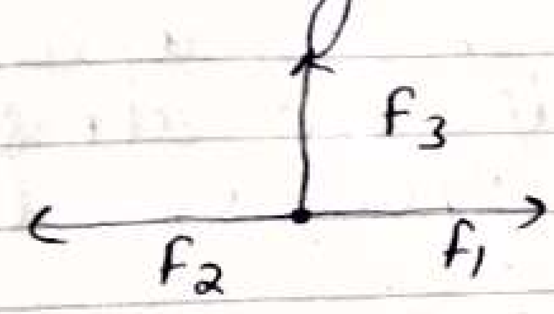
* Opposite direction
* $\theta = 180^\circ$

⇒ Colinear.
In a same line.
$\longrightarrow \quad \longrightarrow \quad \longrightarrow$

⇒ Coplanar (In a single plane)

* 2 vectors are always coplanar.
* 3 vectors may be coplanar or may be not. (They may lie or may not lie on a similiar plane).

⇒ Concurrent Vector
* Forces (Acting at same point)



⇒ Zero vector
whose magnitude is 0 and direction is arbitrary. (It can take any direction).

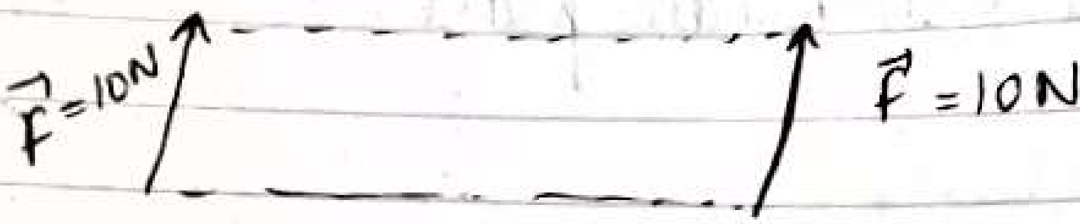
⇒ Unit Vector $[(\hat{A})]$

$\hat{i}$ in x-direction, $\hat{j}$ in y-direction

Magnitude = 1
* It gives direction

$$\hat{A} = \frac{\vec{A}}{|A|} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}}$$

Parallel shift of vector.

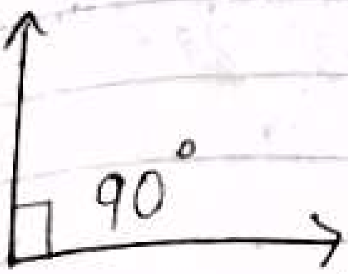


We can shift a vector as it should be parallel shift and the shift should be on the same body

If a force of 10N is applied on a body, it can be shift parallel on the same body.

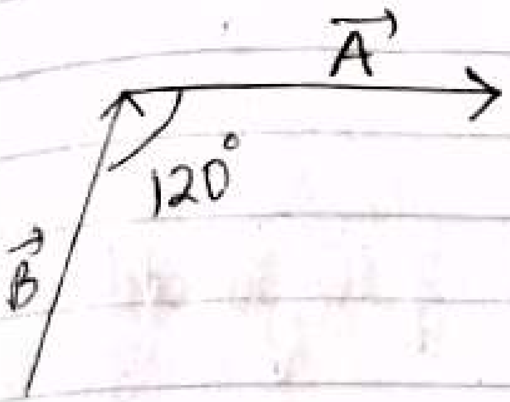
Angles between two vectors (Tail to tail or head to head)

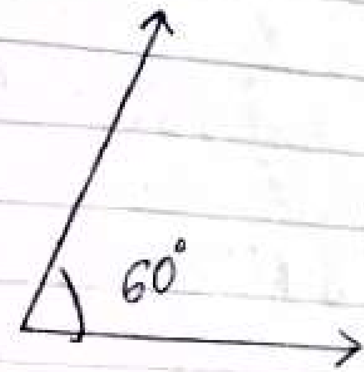
(1)



$\theta = 90^\circ$

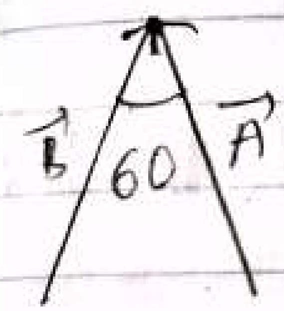
(2)





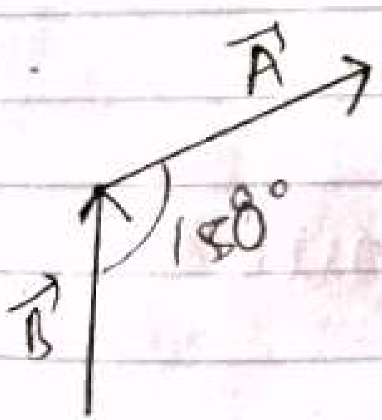
(Parallel shift)

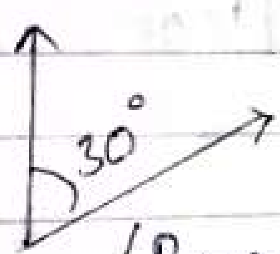
(3)



$\theta = 60^\circ$

(4)

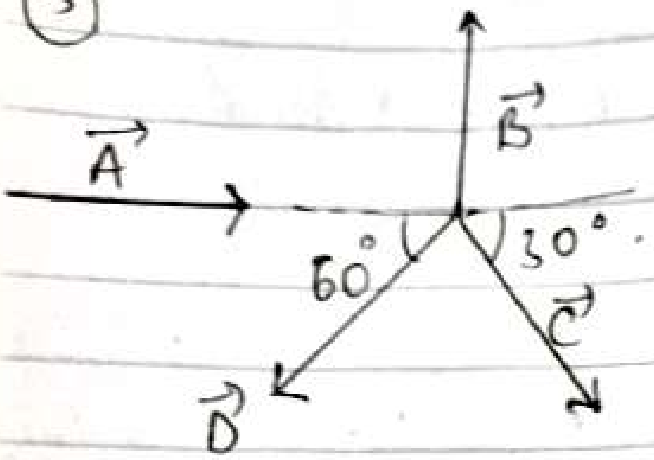


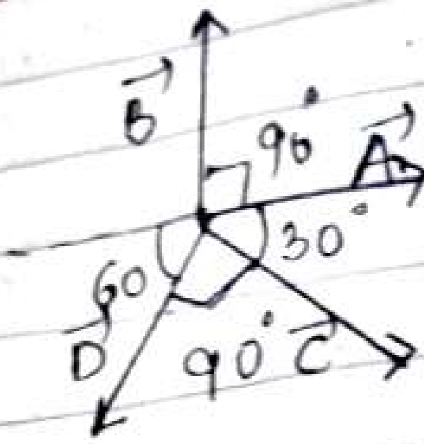


$\theta = 30^\circ$

(Parallel shift)

⑤





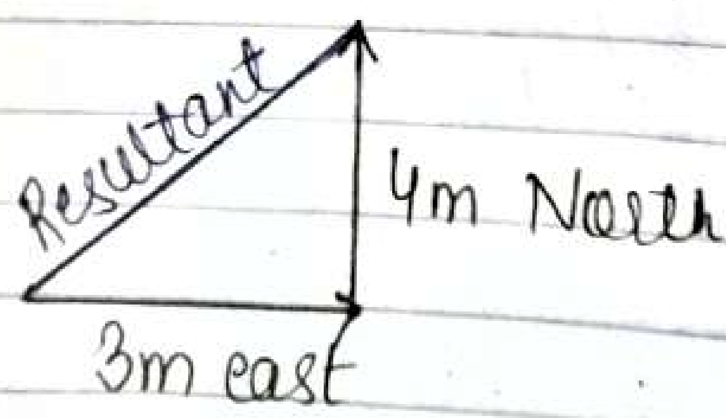
(Tail to Tail)

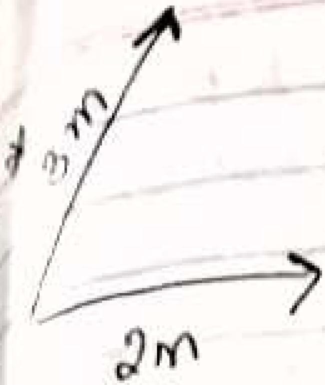
$(0 \leq \theta° \leq 180)$

Smaller angle will be preferred.

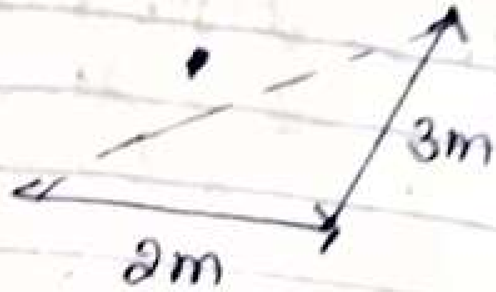
Addition AND SUBTRACTION.

(Head-Tail Method).

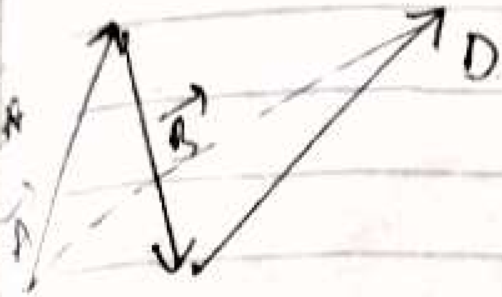




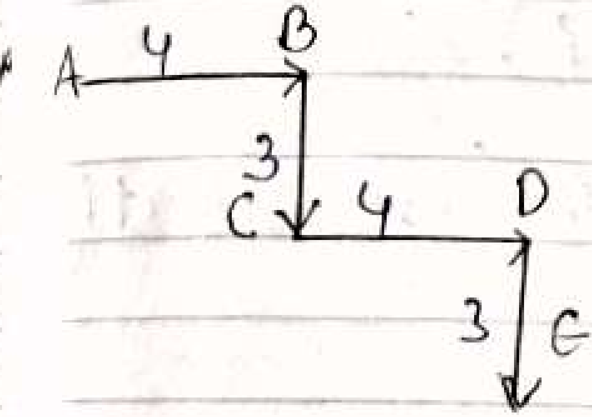
$\Rightarrow$



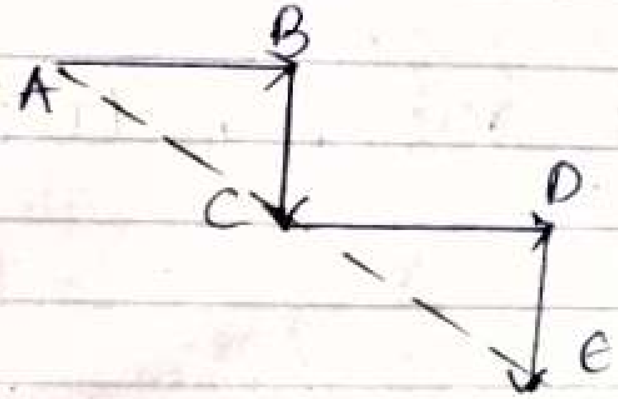
(Parallel shifting)



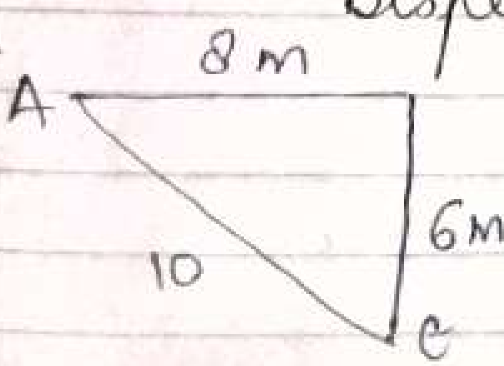
Resultant = Displacement = I^{st} vector tail + last vector head



Displacement = ?

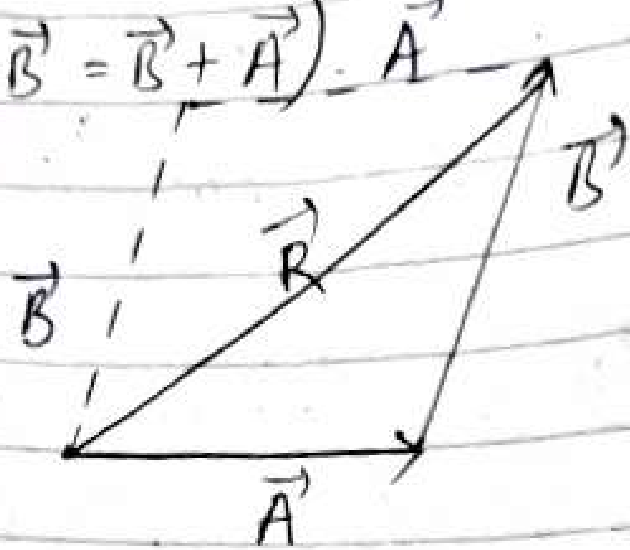


Displacement AE.



Is Vector Commutative $(\vec{A}+\vec{B}=\vec{B}+\vec{A})$

$\Rightarrow (\vec{A}+\vec{B}=\vec{B}+\vec{A})$



We know that,

$$\vec{A}+\vec{B}=\vec{R} \quad \text{—(i)}$$

$$\vec{B}+\vec{A}=\vec{R} \quad \text{—(ii)}$$ (Parallel shifting)

So, $\vec{A}+\vec{B}=\vec{B}+\vec{A}$

Hence vectors are commutative

⇒ Vectors can not be added as all scalar quantities are added.

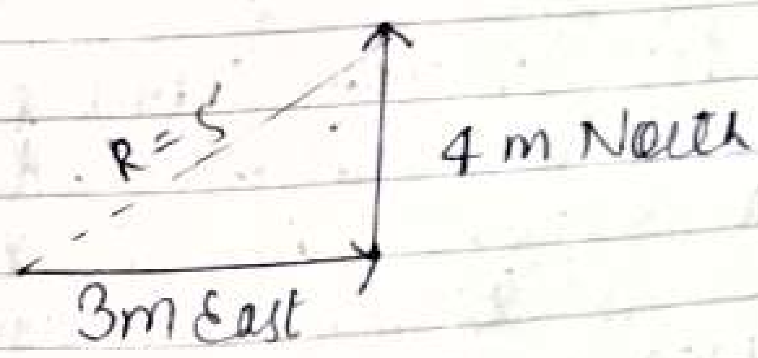
VECTOR ADDITION.

1. Head-Tail Method.
2. Parallelogram Law.
3. Triangle Law.

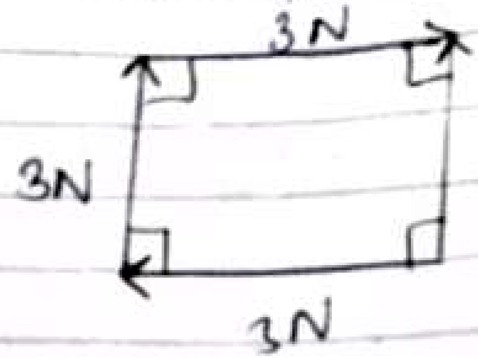
1. Head-tail method

Join tail of next vector with Head of previous vector

3m East + 4m North = R



⇒ Add 3 vectors
3N West, 3N North, 3N East



Resultant

$\vec{R} = 3\ [90°]$

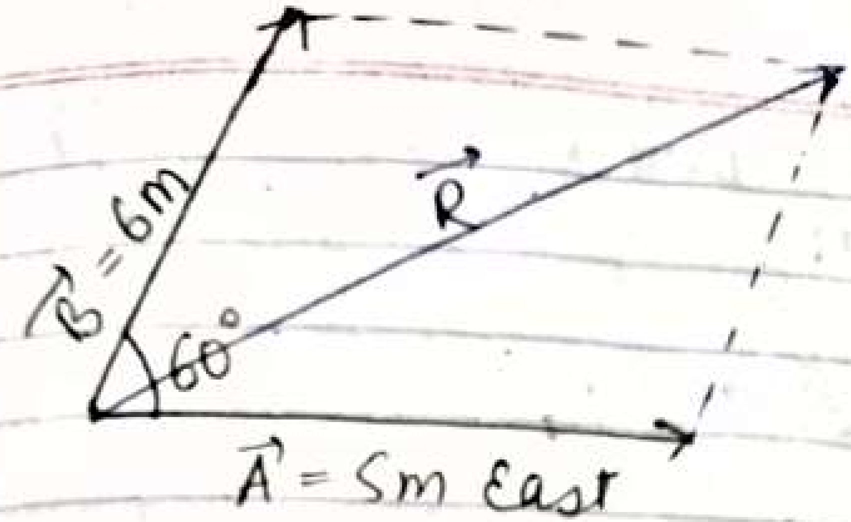
⇒ 5m east then 5m at 60' from east

This law fails here

<u>Parallelogram Law</u>

It allows us to add any kind of vector.

⇒ Join two vectors from tail to tail as the two adjacent sides of parallelogram., $\vec{A} = 5$ East,
$\vec{B}$, 6m, 60° from East.
(Imagine complete ||gm)



$\vec{R}$ = diagonal of ||gm from common point.

$\vec{R} = \vec{A} + \vec{B}$.

$R^2 = A^2 + B^2 + 2AB\cos\theta$. (Magnitude)

θ = angle between 2 vectors

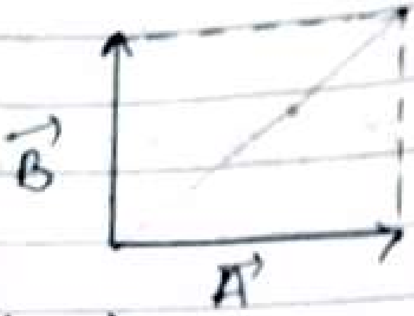
$R^2 = 5^2 + 6^2 + 2 \times 5 \times 6 \times \frac{1}{2}$

$R^2 = 25 + 36 + 30$.

$R = 9$.

Quest- Add two vectors 6 units, 8 units at 90°

$\vec{A} = 6$, $\vec{B} = 8$, $\theta = 90°$

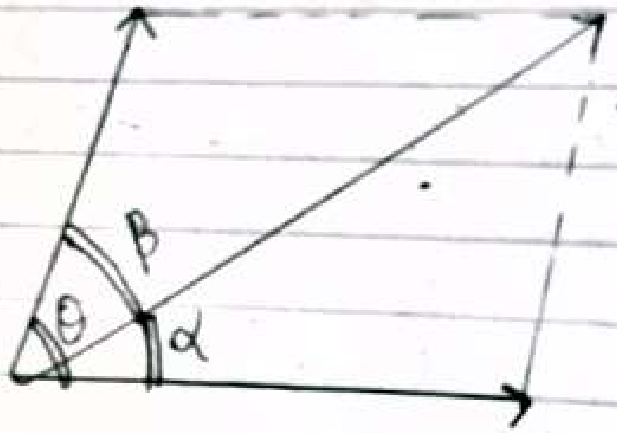


$\vec{R} = \vec{A} + \vec{B}$

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$R^2 = 36 + 64 + 2 \times 6 \times 8 \times 0$

$R = \pm 10$



$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}.$$

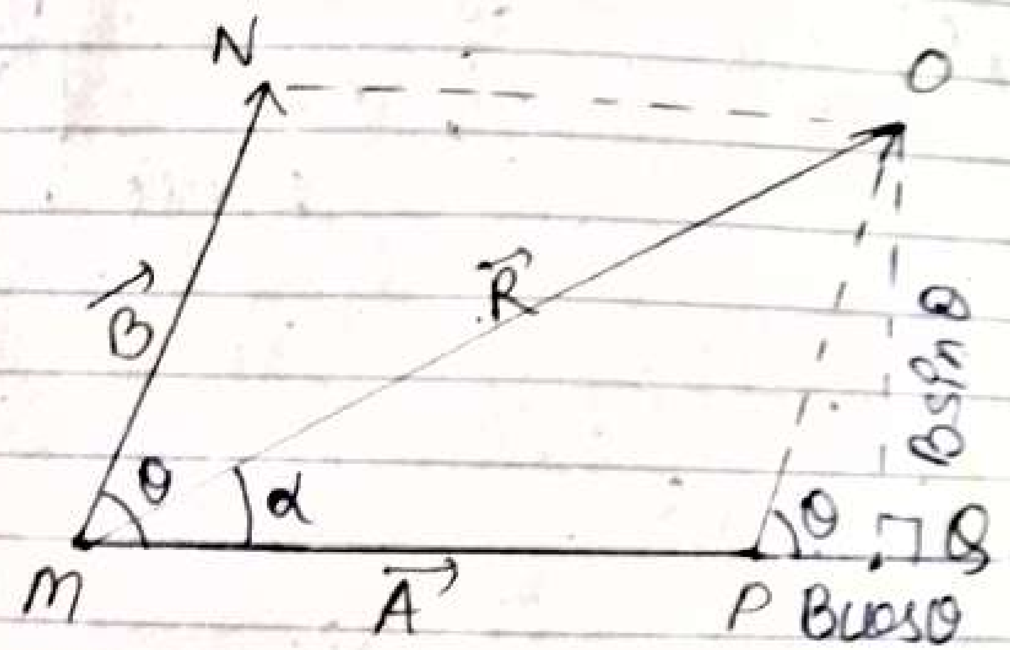
(Direction of resultant)

$\tan\beta = \dfrac{A\sin\theta}{B + A\cos\theta}$ $(\beta = \theta - \alpha)$

$\vec{R}$ direction is from vector $\vec{A}$ (α)

$\vec{R}$ direction from $\vec{B}$ (β)

Derive. $R^2 = A^2 + B^2 + 2AB\cos\theta$



Parallelogram's pair of opp sides is parallel and equal.

ΔPOQ

$\cos\theta = \dfrac{B}{H}$

$\cos\theta = \frac{PQ}{PO}$

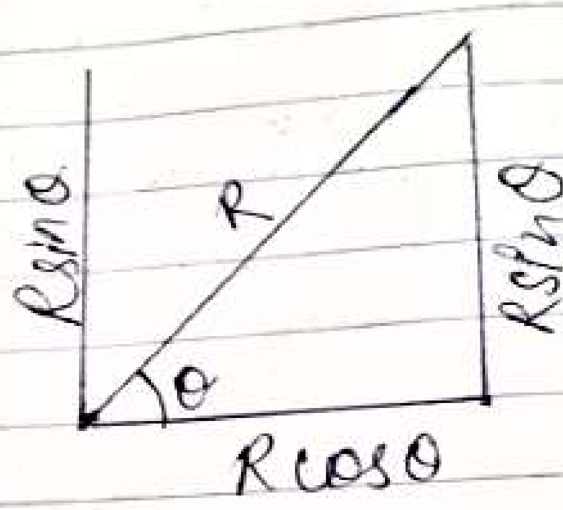
$PQ = PO\cos\theta$

$\boxed{PQ = B\cos\theta}$

$\sin\theta = \frac{P}{H}$

$\sin\theta = \frac{OQ}{PO} = \frac{OQ}{B}$

$\boxed{OQ = B\sin\theta}$



In ΔOQM

$(OM)^2 = (OQ)^2 + (MQ)^2$

$R^2 = (B\sin\theta)^2 + (A + B\cos\theta)^2$

$R^2 = B^2\sin^2\theta + A^2 + B^2\cos^2\theta + 2AB\cos\theta$

$R^2 = A^2 + B^2\sin^2\theta + B^2\cos^2\theta + 2AB\cos\theta$

$R^2 = A^2 + B^2(\sin^2\theta + \cos^2\theta) + 2AB\cos\theta$

$$\boxed{R^2 = A^2 + B^2 + 2AB\cos\theta}$$

Direction of Resultant

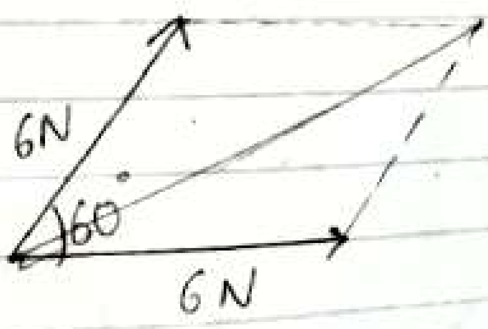
In ΔOQM

$$\tan\alpha = \frac{P}{B}$$

$$\tan\alpha = \frac{OQ}{MQ} = \frac{B\sin\theta}{A+B\cos\theta}$$

$$\boxed{\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}}$$

Ques6- Two forces of magnitude 6N each at a point as shown. Find the resultant.



$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$R^2 = 36 + 36 + 2 \times 36 \cos 60^\circ$$

$$R^2 = 108$$

$$R = 6\sqrt{3}$$

$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$

$\tan\alpha = \frac{1}{\sqrt{3}}, \alpha = 30°$

{If 2 vectors are equal in magnitude the resultant will pass through the angle between them.}

Quesb- Two vectors of equal magnitude are added to give resultant, which is of same magnitude as the 2 vector. Find the angle between them.

$R = A = B = x$

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$x^2 = x^2 + x^2 + 2x^2\cos\theta$

$-x^2 = 2x^2\cos\theta$

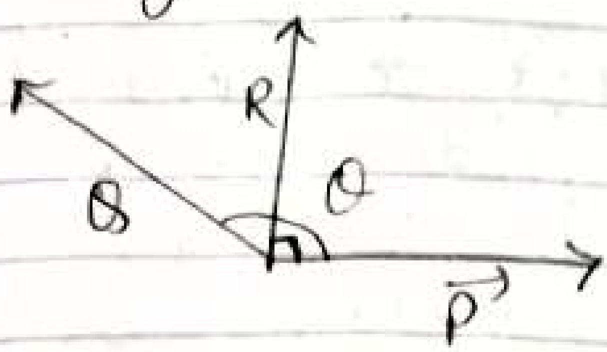
$\cos\theta = \frac{-x^2}{2x^2} = \frac{-1}{2}$

$\cos\theta = \frac{-1}{2}$

$\boxed{\theta = 120°}$

Quesb- Two vectors P(smaller one) & Q as a sum of 18 and there resultant is 12. The resultant is to smaller of two vector. Find the value of P & Q and angle between them.

$P + Q = 18$
$P' + Q' = 12$



$R^2 = A^2 + B^2 + 2AB\cos\theta$
$12^2 = P^2 + Q^2 + 2PQ\cos\theta$.

$12^2 = P^2 + Q^2 + 2P(-P)$
$12^2 = P^2 + Q^2 - 2P^2$.
$12^2 = Q^2 - P^2$

$(Q - P)(Q + P) = 144$
$Q - P(18) = 144$
$Q - P = 8$
$+\ P + Q = 18$
$2Q = 26,\ \boxed{Q = 13}$

$12^2 = 13^2 - P^2$
$\boxed{P = 5}$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\tan 90^\circ = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$P + Q\cos\theta = 0$
$Q\cos\theta = -P$.

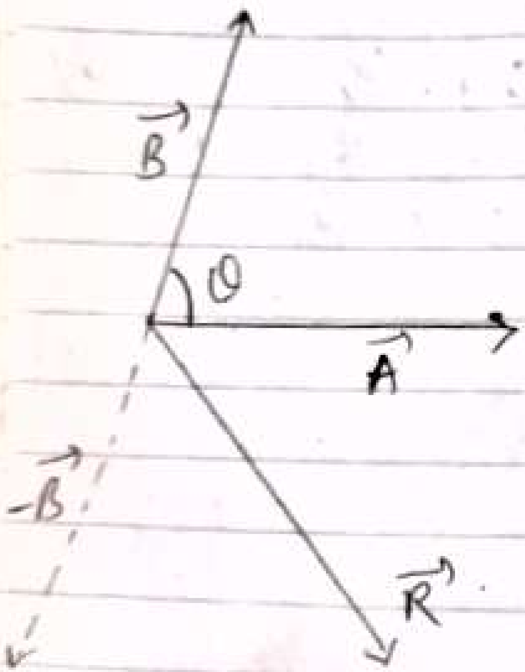
$Q\cos\theta = -5$
$$\boxed{\cos\theta = -\frac{5}{13}}$$

Subtraction of Vector

(Means negative of vector (opp in direction

⇒ Vectors can only be added.

⇒ $\vec{A} - \vec{B}$ (×)
⇒ $\vec{A} + (-\vec{B})$ (✓)



$\vec{R} = \vec{A} + (-\vec{B})$

angle $= (180 - \theta)$

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$R^2 = A^2 + B^2 + 2AB\cos(180 - \theta)$

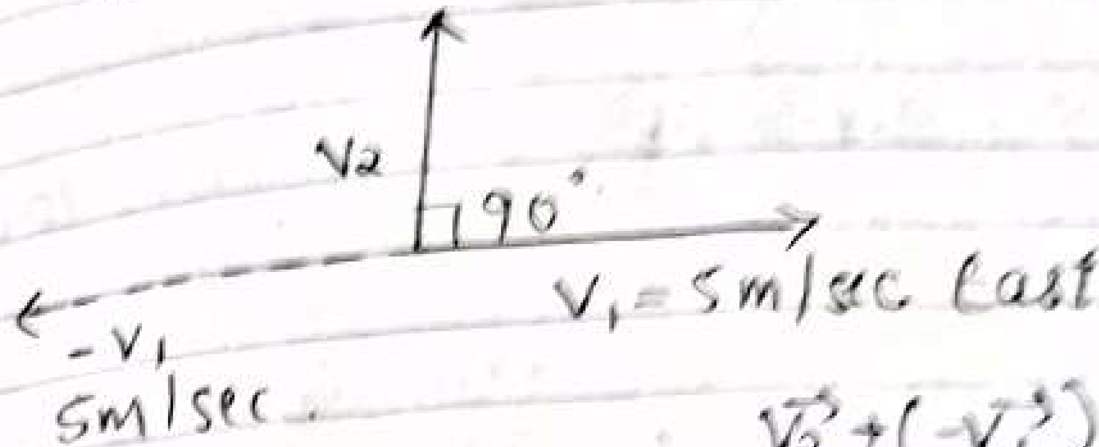
$R^2 = A^2 + B^2 + 2AB(-\cos\theta)$

$\cos(180 - \theta) = -\cos\theta$

$$\boxed{R^2 = A^2 + B^2 - 2AB\cos\theta}$$

Ques6- A car runs at 5m/sec East, take a sharp turn to North and continues at 5m/sec. Find the change in velocity of car.

Δv = change in velocity $\Rightarrow \vec{v_2} - \vec{v_1}$

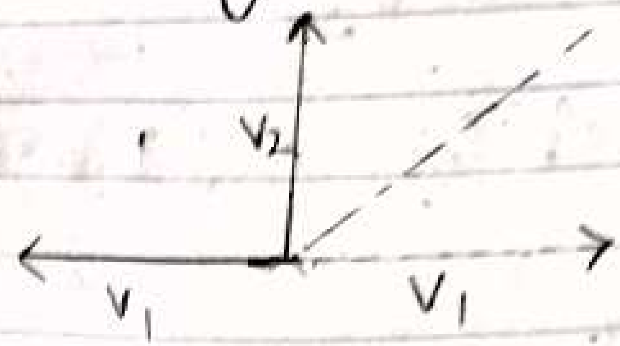


$\vec{v_2} + (-\vec{v_1})$

$R^2 = A^2 + B^2 - 2AB\cos 90°$

$[R = 5\sqrt{2}]$ North west.

Ques8- A car running at 10m/sec (west) takes a sharp turn towards north and continues at 10m/sec. If it takes 2 sec in turning. Find acc of car.



$\vec{a} = \dfrac{\vec{v_2} - \vec{v_1}}{t}.$

$R^2 = A^2 + B^2 - 2AB\cos\theta$
$R^2 = 200 - 200 \times 0$
$R = 10\sqrt{2}$ NE

$a = \frac{\Delta v}{t} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$ NE m/sec^2.

Ques6- A plane moving with velocity v turns by 'θ' angle & its speed remains 'v'. find the change in velocity of plane.

$\vec{V_2} - \vec{V_1}$

Ans:- $(R)^2 = A^2 + B^2 - 2AB\cos\theta$
$(R)^2 = v^2 + v^2 - 2v \times v\cos\theta$.
$(R)^2 = 2v^2 - 2v^2\cos\theta$
$(R)^2 = 2v^2(1 - \cos\theta)$
$(R)^2 = 2v^2 \cdot 2\sin^2\theta/2$.

$(R)^2 = 4v^2\sin^2\theta/2$

$\left\{ 1 - \cos\theta = 2\sin^2\theta/2 \right\}$

$R = 2v\sin\theta/2$.

Quest- The difference of 2 unit vectors is a unit vector. find the angle between 2 vectors.

$\vec{A} = 1$, $\vec{B} = 1$, $R = 1$

$R^2 = A^2 + B^2 - 2AB\cos\theta$.

$1^2 = 1^2 + 1^2 - 2\cos\theta$

$\cos\theta = \frac{1}{2}$

$\theta = 60^\circ$.

Quest- The sum and difference are equal in magnitude. Find the angle btw vectors

$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$.

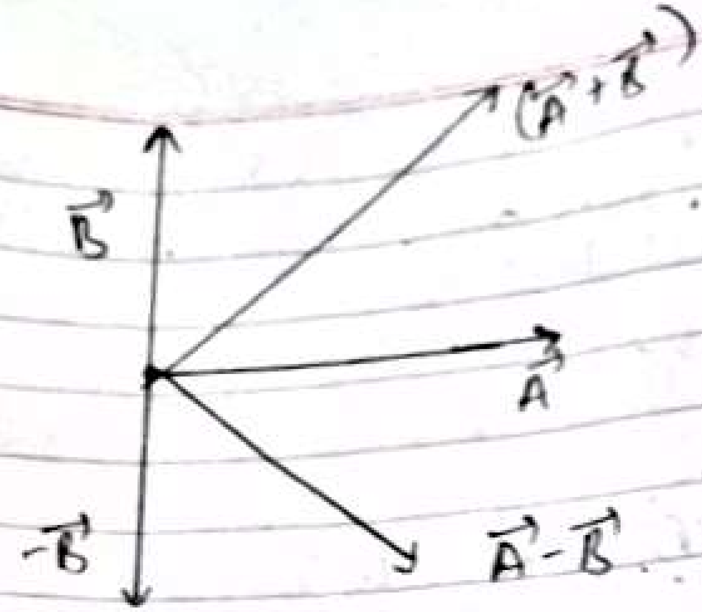
Let $|\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$.

$A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$.

$4AB\cos\theta = 0$.

$\cos\theta = 0$

$\theta = 90^\circ$.



Ques 8 - 3 vectors $\vec{A}+\vec{B}+\vec{C}=0$, if $|\vec{A}|=12$, $|B|=5$, $|C|=13$
Find angle between $\vec{A}$ and $\vec{B}$.

$$|\vec{A}+\vec{B}|^2 = |(-C)|^2$$

$$A^2+B^2+2AB\cos\theta = C^2$$
$$144+25+120\cos\theta = 169$$
$$\cos\theta = 0$$
$$\theta = 90^\circ$$

⇒ We have 2 vectors 3 & 4, Their resultant cannot be
(a) 2
(b) 6
✓(c) 8
(d) 4

Max value of any vector

$R = |A+B|$

Min value of any vector

$R = |A-B|$

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Multiplication of Vector

1. Scalar × Vector
2. Vector × Vector = Scalar
3. Vector × Vector = Vector

$\vec{A} = \underbrace{2\hat{i} - \hat{j} + \hat{k}}$ Cartesian form

$3\vec{A} = 6\hat{i} - 3\hat{j} + 3\hat{k}$

Vector × Vector = Scalar

$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$
$\vec{B} = 3\hat{i} + 4\hat{j} + 2\hat{k}$

$\vec{A} \cdot \vec{B} = C$ (3, 4, 5 . .)

↓ Scalar

⇓ (dot product)

V × V = V

$\vec{A} \times \vec{B} = \vec{C}$ (vector product)

⇓

(cross product).

(22) * If the angle between A & B are greater than 90°, the dot product will be -ive

(vector ~~will~~ length will always be +ive)

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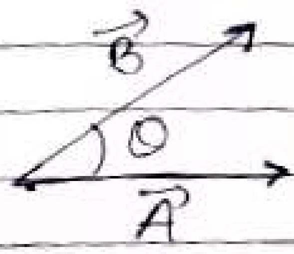
Dot Product

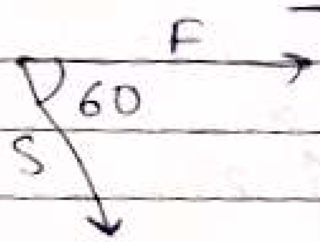
$\vec{A} \cdot \vec{B} = C.$

Work = $\vec{F} \cdot \vec{S}$



$\vec{A} \cdot \vec{B} = |A| \times |B| \times \cos\theta$.

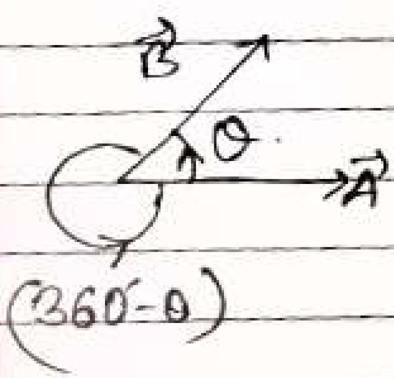
$\vec{F} = \underline{10N}$ $\vec{S} = \underline{5m}$



Work = $\vec{F} \cdot \vec{S}$

Work = 25 J

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$.



(A se B angle)

$\vec{A} \cdot \vec{B} = |A| |B| \cos\theta$.

$\vec{B} \cdot \vec{A} = |B| |A| \cos(360° - \theta)$

(B se A angle)

$\cos(360° - \theta)$
$= \cos\theta$

* If $\vec{A} \perp \vec{B}$ $\theta = 90°$

$\vec{A} \cdot \vec{B} = |A| |B| \cos 90°$

$\vec{A} \cdot \vec{B} = 0$

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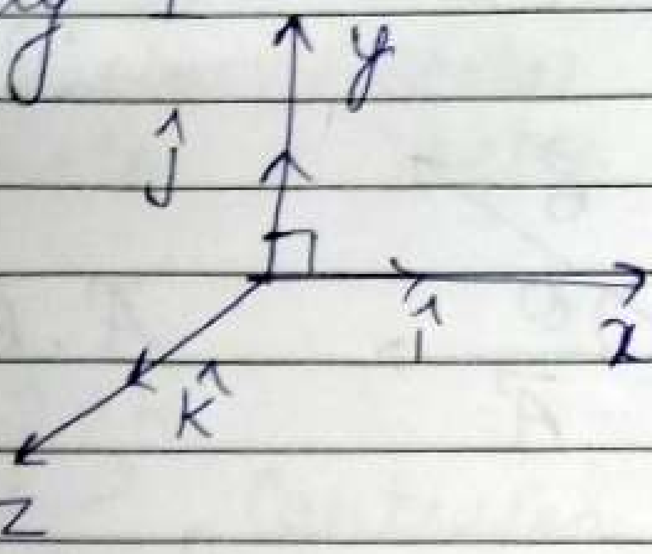
Orthogonal unit vectors

mutually ⊥



$\hat{i}$ = whose mag is 1 and is in the direction of x.

similary $\hat{j}$ & $\hat{k}$

$\hat{i}.\hat{j} = 0$; $\hat{i}.\hat{i} = 1$

$\hat{i}.\hat{k} = 0$; $\hat{j}.\hat{j} = 1$

$\hat{j}.\hat{k} = 0$; $\hat{k}.\hat{k} = 1$

$\hat{i}.\hat{i} = |\hat{i}||\hat{i}| \cos 0°$

$\Rightarrow 1.$

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Quest- $\vec{A} = 2\hat{i} + 3\hat{j}$
$\vec{B} = 4\hat{i} + 5\hat{j}$

find $\vec{A}.\vec{B}$

$\Rightarrow (2\hat{i} + 3\hat{j}).(4\hat{i} + 5\hat{j})$

$\Rightarrow 8(1) + 15(1)$

$\vec{A}.\vec{B} \Rightarrow 23$

$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$
$\Rightarrow 2\times4 + (3\times5) + 0\times0$
$\Rightarrow 23$

Quest- Find, if
$\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$
$\vec{B} = \hat{i} - \hat{j} + 3\hat{k}$

$\vec{A}.\vec{B} = 2 - 1 + 3$
$\vec{A}.\vec{B} \Rightarrow 5$

Quest- If a vector $(2\hat{i} + 3\hat{j} + 8\hat{k})$ is $\perp$ to the vector $4\hat{i} - 4\hat{j} + a\hat{k}$, then the value of a is.

$\vec{A}.\vec{B} = 0 \quad (\perp)$

$0 = 8 - 12 + 8a$
$0 = -4 + 8a$

$4 = 8a$
$a = \frac{1}{2}$

8-

Angle between two vectors.

$\vec{A}.\vec{B} = |A||B| \cos\theta$

$$\boxed{\cos\theta = \frac{\vec{A}.\vec{B}}{|A||B|}}$$

$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$
$\vec{B} = \hat{i} - \hat{j}$.

$\cos\theta = \frac{\vec{A}.\vec{B}}{|A||B|}$

$\vec{A}.\vec{B} = (2 + 1)$
$\Rightarrow 3$

$|A||B| = \sqrt{A_x^2 + A_y^2 + A_z^2} \sqrt{B_x^2 + B_y^2 + A_z^2}$

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$\Rightarrow \sqrt{4+1+1}\ \sqrt{1+1}$

$\Rightarrow \sqrt{6}\ \sqrt{2}$

$\Rightarrow 2\sqrt{3}\ .\sqrt{12}$

$$\begin{array}{c|c} 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array}$$

$$\boxed{\cos\theta = \frac{3}{2\sqrt{3}}}$$

Quest- $\vec{P} = 2\hat{i} + \hat{j} - \hat{k}$, find θ

$\vec{Q} = \hat{i} - \hat{j}$

$$\cos\theta = \frac{2-1}{\sqrt{6}\sqrt{2}}$$

$$\cos\theta \Rightarrow \frac{1}{2\sqrt{3}}.$$

Quest- $\vec{R} = \hat{i} + \hat{j}$ (find θ).

$\vec{S} = \hat{i} - \hat{j}$

$$\cos\theta = \frac{1-1}{\sqrt{2}\ \sqrt{2}} \Rightarrow \frac{0}{\sqrt{4}} \Rightarrow \frac{0}{2} = 0$$

Quest- Find the angle that $\vec{A} = \hat{i} + \hat{j}$ makes with x axis -

$\vec{A} = \hat{i} + \hat{j}$

$\vec{B} = \hat{i}$ (any vector along x-axis) can be $2\hat{i}$, $0.5\hat{i}$ .. anything

$$\cos\theta = \frac{\vec{A}\,\vec{B}}{|A|\,|B|}$$

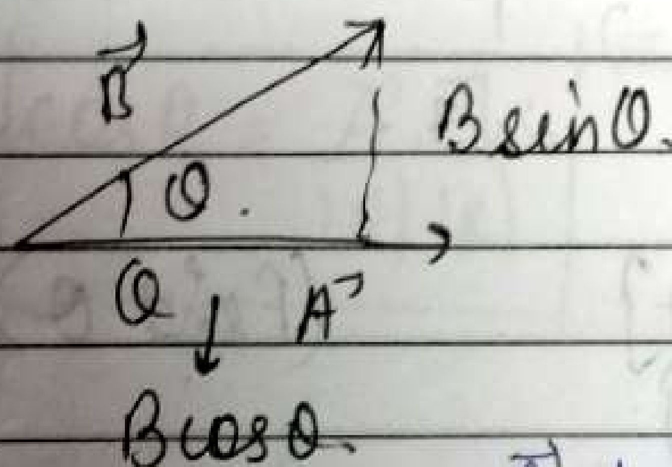
$$\cos\theta \Rightarrow \frac{1\times 1\times 0}{\sqrt{2}\,\sqrt{1}}$$

$$\cos\theta \Rightarrow \frac{1}{\sqrt{2}}$$

$$\boxed{\theta = 45^\circ}$$

#

$$\vec{A}\cdot\vec{B} = |A|\,|B|\cos\theta$$



$\vec{B}$ projection on $\vec{A}$

$B\cos\theta$ is along $\vec{A}$.

$$\vec{A}\cdot\vec{B} = |\vec{A}|\,(|\vec{B}|\cos\theta)$$

Cross / Vector Product

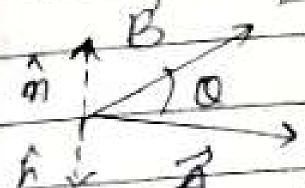
$\vec{A} \times \vec{B} = \vec{C}$ vector

Torque / moment of force $= \vec{F} \times$ displacement

(This is $\perp$ to $\vec{A}$ & $\vec{B}$)

$\vec{A} \times \vec{B} = \boxed{|A||B|\sin\theta} \, \hat{n}$

$\hat{n}$ → direction

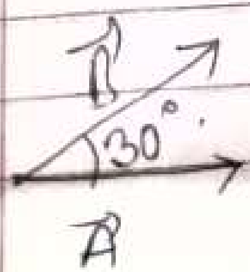


To give direction to vectors we use unit vector

$\vec{C} \perp \vec{A}$
$\vec{C} \perp \vec{B}$

Ques) $\vec{A} = 5$, $\vec{B} = 2$, $\theta = 30°$, find $\vec{A} \times \vec{B} = |A||B|\sin\theta$
$\vec{B} \times \vec{A} =$

$\vec{A} \times \vec{B} = 5$
$\vec{B} \times \vec{A} = |B||A|\sin\theta = 5$



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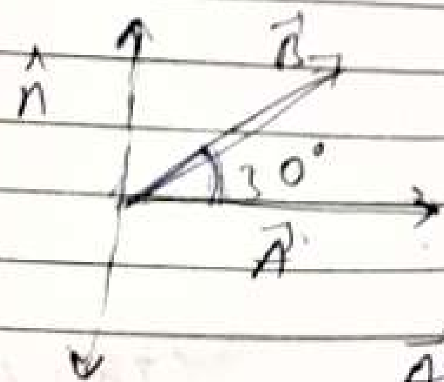
$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$



(Right hand thumb rule)

$\vec{A} \times \vec{B}$ = while curling from $\vec{A}$ to $\vec{B}$ using R.H. Thumb rule the thumb is upward so the $\hat{n}$ will be upward.

$\vec{B} \times \vec{A}$ = while curling from $\vec{B}$ to $\vec{A}$ using R.H. Thumb rule the thumb will be downward

(as ~~we~~ we will take the smaller angle between the vectors).

$\vec{A} \times \vec{B}$ = 5 upwards
$\vec{B} \times \vec{A}$ = 5 downwards

So, $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

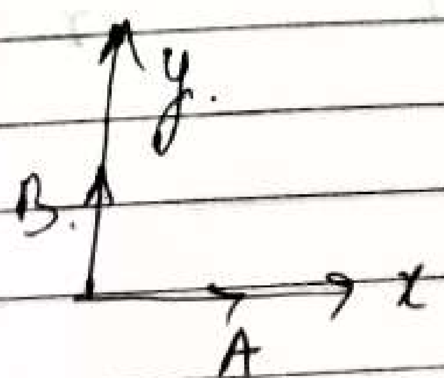
OR Screw Rule.

(Add both the vector tail to tail ; move the screw from $\vec{A}$ to $\vec{B}$, so the direction is upwards)

and move the screw from $\vec{B}$ to $\vec{A}$ so the screw will go downwards.

commutative rule is not valid for cross product.

Quest→



$\vec{A} \times \vec{B}$

what will be the direction of $\vec{A} \times \vec{B}$

upwards (outwards)

$\vec{B} \times \vec{A}$ (inwards) downwards

Orthogonal unit vector:

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

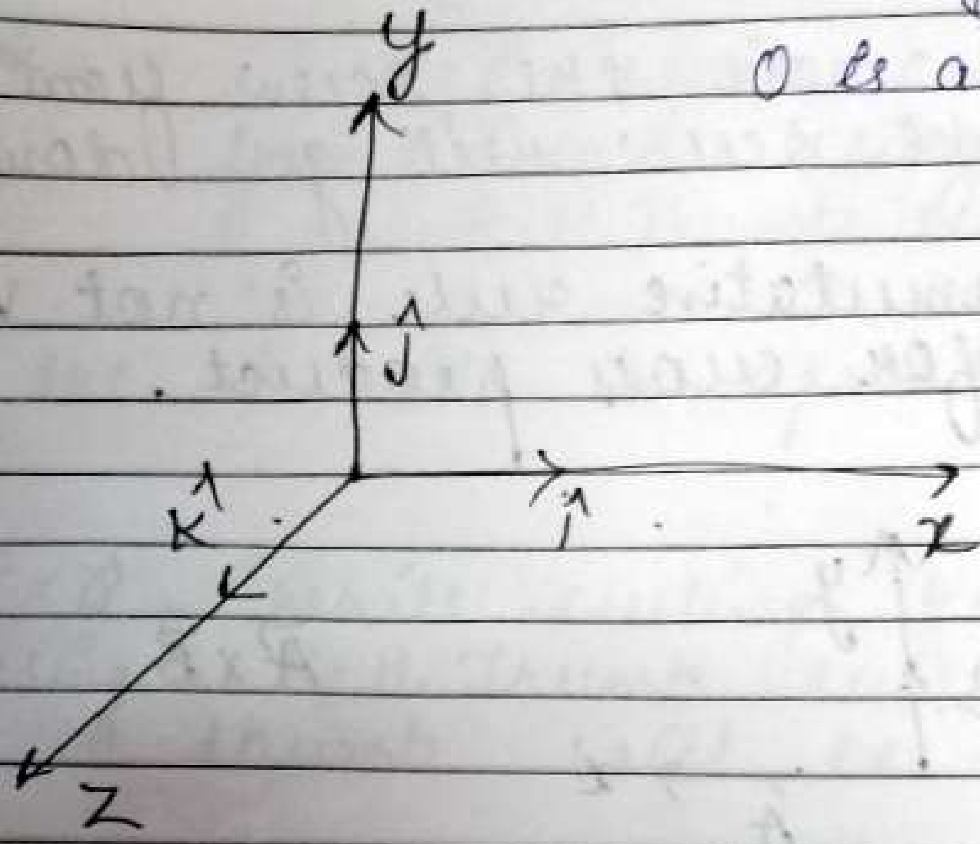
$\hat{i} \times \hat{i} = |\hat{i}||\hat{i}|\sin 0 = \vec{0}$

$\hat{j} \times \hat{j} = |\hat{j}||\hat{j}|\sin 0 = \vec{0}$

$\hat{k} \times \hat{k} = \vec{0}$

$\Downarrow \qquad \vec{0}$

0 is a vector



$\hat{i} \times \hat{j} = |\hat{i}||\hat{j}| \sin 90^\circ$

$\Rightarrow 1 \; \hat{n} \longrightarrow$ (unit vector outwards).

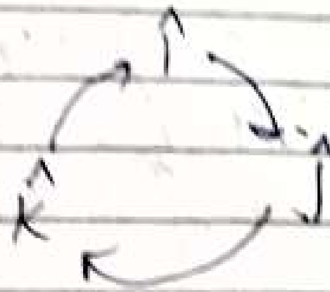
$\Rightarrow \hat{k}$

$\hat{k} \times \hat{i} = \hat{j}$

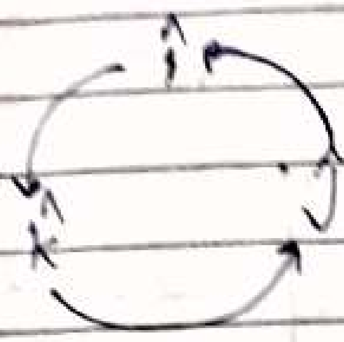
$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{i} \times \hat{k} = -\hat{j}$



clockwise = +ive



anticlockwise = -ive

Ques: $\vec{A} = 5\hat{i}$

$\vec{B} = 2\hat{k}$

$\vec{A} \times \vec{B} = 5\hat{i} \times 2\hat{k}$

$= -10\hat{j}$

$\vec{B} \times \vec{A} = 10\hat{j}$

$$\boxed{\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}}$$

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Ques6- $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.
$\vec{B} = 3\hat{i} + 2\hat{j} + 3\hat{k}$.

$\vec{A} \times \vec{B} = 6(0) + 4\hat{k} + 6(-\hat{j})$
$9(-\hat{k}) + 6(0) + 9\hat{i}$
$12\hat{j} + 8(-\hat{i}) + 12(0)$

$\vec{A} \times \vec{B} = \hat{i} + 6\hat{j} - 5\hat{k}$.

Short cut.

$$\begin{array}{c} \\ A \\ B \end{array}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 2 & 3 \end{vmatrix}$$

$\vec{A} \times \vec{B} = \hat{i}(9-8) - \hat{j}(6-12) + \hat{k}(4-9)$

$\vec{A} \times \vec{B} = 1\hat{i} - (-6)\hat{j} + (-5\hat{k})$

$\boxed{\vec{A} \times \vec{B} = \hat{i} + 6\hat{j} - 5\hat{k}}$

Ques6- Find the mag. of $\vec{A} \times \vec{B}$ if $A = 2\hat{i} + \hat{j} - \hat{k}$
and $B = 6\hat{i} + 3\hat{j} - 3\hat{k}$

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$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 6 & 3 & -3 \end{vmatrix}$$

$\Rightarrow \hat{i}(-3+3) - \hat{j}(-6+6) + \hat{k}(6-6)$

$\Rightarrow \hat{i}(0) - \hat{j}(0) + \hat{k}(0)$

$\Rightarrow 0$

$|\vec{A} \times \vec{B}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Mag:- If $\vec{A} \times \vec{B} = (3\hat{i} + 2\hat{j} + 4\hat{k})$

$|\vec{A} \times \vec{B}| = \sqrt{3^2 + 2^2 + 4^2}$ (Mag)

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* If $\vec{A} \times \vec{B} = 0$

Either $A = 0$ or $B = 0$

or

$|A||B| \sin\theta = 0$

$\sin\theta = 0$

$\theta = 0^\circ$

$\vec{A} \cdot \vec{B} = 0$

$\theta = \perp$ or 90°

$\vec{A} \times \vec{B} = 0$

$\theta = 0^\circ$

Quest- $|\vec{A}| = 5$ (Mag)

$|B| = 6$.

$|\vec{A} \times \vec{B}| = 15$ (Mag)

find angle b/w A & B

$|\vec{A} \times \vec{B}| = |A||B| \sin\theta$.

$15 = 5 \times 6 \sin\theta$

$\sin\theta = \frac{1}{2}$, $\theta = 30^\circ$ and 150°

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Find angle between $\vec{A}$ & $\vec{B}$

$|\vec{A}\times\vec{B}| = |A.B|$

$|A||B|\sin\theta = |A||B|\cos\theta$

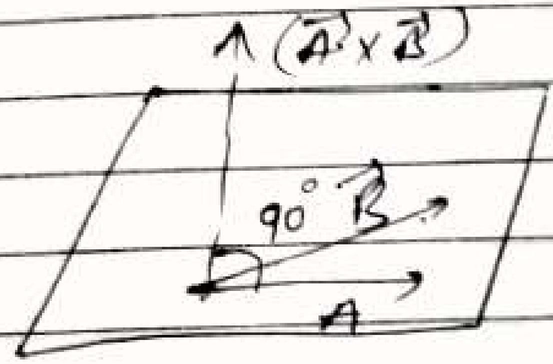
$\frac{\sin\theta}{\cos\theta} = 1$

$\tan\theta = 1$

$\boxed{\theta = 45^\circ}$ or $\frac{\pi}{4}$ → 180°

$A.(\vec{A}\times\vec{B}) = ?$

$\vec{A}.(\vec{A}\times\vec{B})$



$\vec{A}.(\vec{A}\times\vec{B}) = 0$

$(\perp)$ to $(\vec{A}\times\vec{B})$

If $\vec{A}.\vec{B} = 0$

$\vec{A}.\vec{C} = 0$

then $\vec{A}$ is $\parallel$ to

(a) $\vec{C}$ (c) $\vec{B}\times\vec{C}$

(b) $\vec{B}$ (d) B.C

* Vectors are which follows ||gm law

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Unit Vectors

eg:- current is not vector,

→ Magnitude = 1

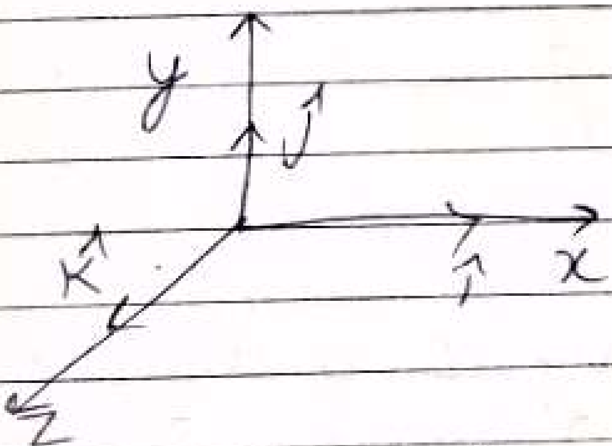
⇛ It gives direction

$\vec{A}$ = Magnitude × direction

$\vec{A} = |A| \times \hat{A}$

$\hat{A} = \frac{\vec{A}}{|A|}$

Orthogonal unit vector



Ques:- A force 10 N is in x direction. Represent it in vector form

$\vec{F} = 10\hat{i}$

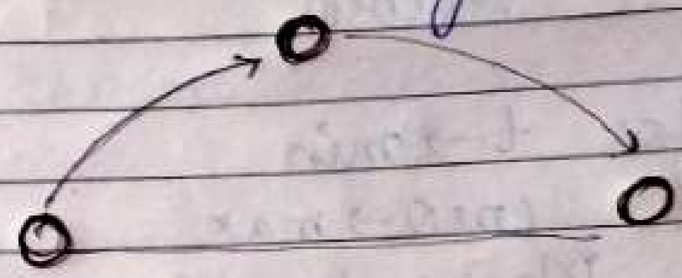
$\boxed{\vec{F} = 10\hat{i}}$

PROJECTILE MOTION

* Actual Meaning of projectile motion is motion under gravity.

* And in syllabus it is Motion in a plane (2-D)

eg:- When a ball is thrown upward with some angle.

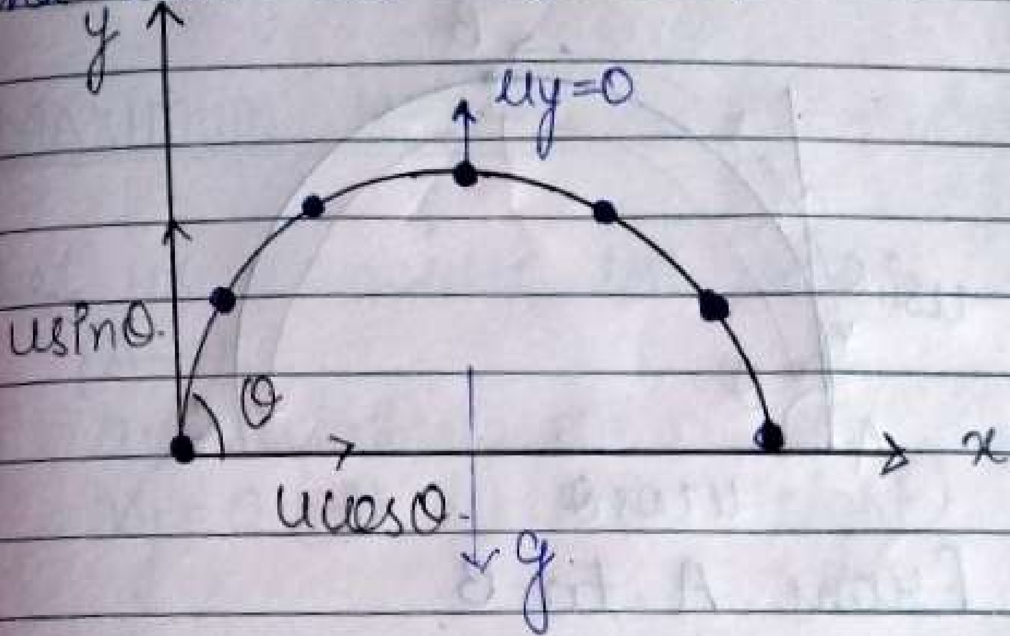


In a projectile Motion,
u is the initial velocity,
θ is the angle of projection,
the motion of an object is in 2 direction, so
Initial velocity in x direction is given by $u_x = u\cos\theta$,
Initial velocity of y direction is given by $u_y = u\sin\theta$.

$\Rightarrow$ Acc. due to gravity in x-direction.
$a_x = 0$

$\Rightarrow$ Acc. due to gravity in y-direction
$a_y = -g$.

so, x direction velocity is always constant whereas, velocity of y direction first increases, and at highest point it becomes 0 and then it increases.



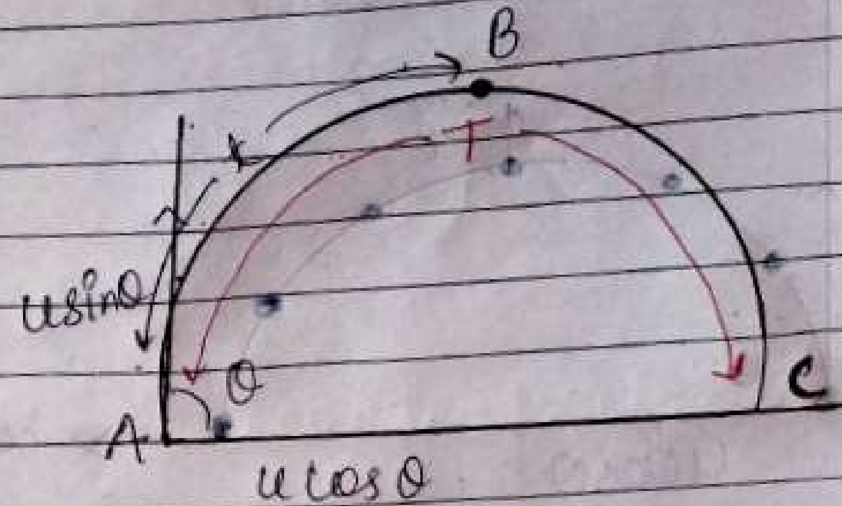
Assuming there is no air resistance, and no friction

In Projectile Motion

⇒ Time of flight is given by **T**
⇒ Max. Height is given by **H**
⇒ Horizontal Range is given by **R**

Time of flight $\left(T = \frac{2u\sin\theta}{g}\right)$

Let us consider motion in y-direction



From A to B.

$u_y = u\sin\theta$,
& we know $a_y = -g$, and

$v_y = 0$ (at B)

Let the object took time (t).

From first equation of motion

$$v = u + at$$
$$0 = u\sin\theta + (-g)\times t$$

$\left[t = \frac{u\sin\theta}{g}\right]$ upto B.

Total time = T

$T = 2t$

$$\boxed{T = \frac{2u\sin\theta}{g}}$$ Time of flight

MAXIMUM HEIGHT $\left(H = \frac{u^2\sin^2\theta}{2g}\right)$

Let us consider the motion in y-direction ($A \to B$).

$u_y = u\sin\theta$

$v_y = 0$ (At B) (Max Height)

$a_y = -g$

Displacement in Y-direction = H

Using 3rd equation of motion

$v^2 = u^2 + 2as$

$0^2 = (u\sin\theta)^2 + 2(-g)H$

$2gH = u^2\sin^2\theta$

$$\boxed{H = \frac{u^2\sin^2\theta}{2g}}$$

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HORIZONTAL RANGE $\left(R = \frac{u^2 \sin 2\theta}{g}\right)$

Let us consider the motion in x-direction (A→C)

$u_x = u\cos\theta$

$a_x = 0$

$s_x = R$

$t = T$

Using second equation of motion

$$s = ut + \frac{1}{2}at^2$$

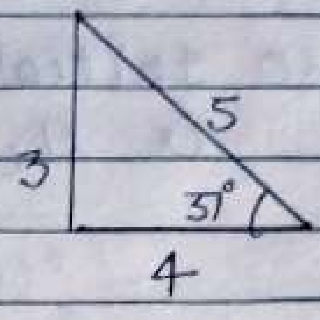
$$R = u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$\boxed{R = \frac{u^2 2\sin\theta\cos\theta}{g}}$$

OR

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

Ques 1- A ball is thrown with 5 m/sec at an angle of projection 37°. Find
(i) Time of flight.
(ii) Max. Height
(iii) Horizontal Range

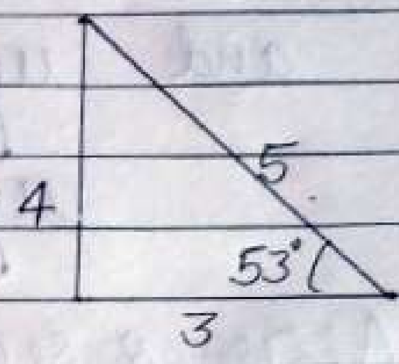


Ans (i) $T = \dfrac{2u\sin\theta}{g}$

$\theta \to 37°$

$T = \dfrac{2 \times 5 \times \sin 37°}{10}$

$T = \dfrac{3}{5} = 0.6$ sec.



(ii) $H = \dfrac{u^2\sin^2\theta}{2g}$

$H = \dfrac{25(\sin 37°)^2}{2 \times 10}$

$H = \dfrac{9}{20}$

(iii) $\text{Range} = \dfrac{u^2\sin 2\theta}{g} = \dfrac{u^2 2\sin\theta\cos\theta}{g}$

$R = \dfrac{12}{5}\,m = 2.4\,m$

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Ques6- A ball is thrown with 50m/sec at angle of projection 37°. Find velocity vector and speed of particle after 2 sec of projection.

Ans6- so, Initial velocity in x direction is $u_x = u\cos\theta$

$= 50 \times \frac{4}{5}$

$u_x = 40$ m/sec;

and $u_y = u\sin\theta$

$u_y = 30$ m/sec.

After 2 sec,

Velocity ~~speed~~ of particle in x-direction is $v_x = 40$ m/sec.

Velocity ~~speed~~ of particle in y-direction is v_y

$u_y = 30$ m/sec.

$a_y = -10$

$t = 2$ sec.

$v_y = u_y + at$

$v_y = 30 + (-10)2$

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$v_y = 30-20$

$v_y = 10\ m/sec$

In vector form

$$\vec{v} = 40\hat{j} + 10\hat{j}$$

speed $|v| = \sqrt{(40)^2+(10)^2}$

$= \sqrt{1600+100}$

$|v| = \sqrt{1700}$

Ques 6- For a projectile motion from ground to ground, H=R, find angle of projection

Ans:-

$$H = R$$

$$\frac{u^2\sin^2\theta}{2g} = \frac{u^2\sin 2\theta}{g}$$

$$\frac{\sin^2\theta}{2} = \sin 2\theta$$

$$\frac{\sin^2\theta}{2} = 2\sin\theta\cos\theta$$

$$\frac{\sin\theta}{2} = 2\cos\theta$$

$$\tan\theta = 4$$

$$\theta = \tan^{-1}4$$

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Ques8- For a projectile motion from ground to ground. If Max. Ht is $30\sqrt{3}$ m and Horizontal Range is 120m. Find initial velocity and angle of projection (u, θ)

Ans :-

$$H = 30\sqrt{3}$$
$$R = 120.$$

$$\frac{u^2 \sin^2\theta}{2g} = 30\sqrt{3} \quad \text{—— (i)}$$

$$\frac{u^2 \sin 2\theta}{g} = 120 \quad \text{or} \quad \frac{2u^2 \sin\theta\cos\theta}{g} = 120 \quad \text{—— (ii)}$$

Dividing eq (i) by (ii) we get

$$\frac{u^2 \sin^2\theta}{2g} \times \frac{g}{2u^2 \sin\theta\cos\theta} = \frac{30\sqrt{3}}{120}.$$

$$\frac{\sin\theta}{4\cos\theta} = \frac{30\sqrt{3}}{120}$$

$$\tan\theta = \sqrt{3}.$$

$$\boxed{\theta = 60^\circ}$$

$$\frac{u^2 \sin^2\theta}{2g} = 30\sqrt{3}$$

$$\frac{u^2 \sin^2 60^\circ}{2g} = 30\sqrt{3}$$

$$\frac{u^2\, 3}{2\times 10\times 4} = 30\sqrt{3}$$

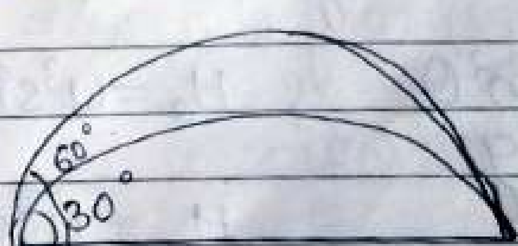
$$u^2 = \frac{80\times 30\sqrt{3}}{3}$$

$$u^2 = \frac{2400}{\sqrt{3}}$$

$$u = \sqrt{\frac{2400}{\sqrt{3}}}$$

RANGE: Range is same for two different angles of projection if u is same

θ°, '$90-\theta$', suppose angles are 30° and 60°.



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PROOF

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}} \quad (\text{For } \theta)$$

For $(90-\theta)$

$$R = \frac{u^2 \sin 2(90-\theta)}{g}$$

$$R = \frac{u^2 \sin(180-2\theta)}{g}$$

$\sin(180-\theta) = \sin\theta$

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

For two angle of projection, Range is same, and vertical max Heights are different i.e. H_1 & H_2. Find relation between H_1, H_2 & R.

For θ, Height is H_1

For $(90-\theta)$, Height is H_2

$$H_1 = \frac{u^2 \sin^2\theta}{2g}, \quad H_2 = \frac{u^2 \sin^2(90-\theta)}{2g}$$

$$H_2 = \frac{u^2 \cos^2\theta}{2g}$$

$$u\sin\theta = \sqrt{2gH_1}$$

$$u\cos\theta = \sqrt{2gH_2}$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2\, 2 \sin\theta \cos\theta}{g}$$

$$= (u \sin\theta)(u\cos\theta) \times \frac{2}{g}$$

$$R = \sqrt{2gH_1}\,\sqrt{2gH_2} \times \frac{2}{g}$$

$$R = 2\sqrt{4H_1H_2}$$

$$R = 4\sqrt{H_1H_2}$$

When Range will be Maximum

$$R = \frac{u^2 \sin 2\theta}{g}$$

Range depends upon speed and θ

If $\theta \to 45^\circ$

$R \to max.$

when $\frac{dR}{d\theta} = 0$ (Range will be Maximum)

$$\frac{dR}{d\theta} = \frac{u^2 \cos 2\theta \times 2}{g}$$

$$\frac{u^2 \cos 2\theta \times 2}{g} = 0$$

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$\cos 2\theta = 0$

$2\theta = 90°$

$\theta = 45°$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin 90}{g}$$

$$\boxed{R_{max} = \frac{u^2}{g}}$$

Max Range is $\frac{u^2}{g}$ when $\theta = 45°$.

Path of projectile motion is parabolic

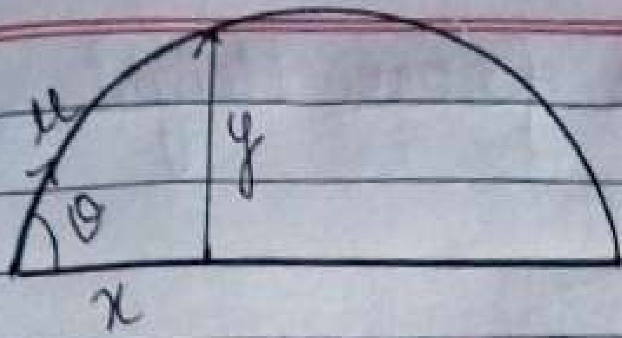
⇒ condition of parabola

$$y^2 \propto x$$

$$y^2 = 4ax$$

When in an equation y & x are relatable that equation is called as equation of trajectory (Path)

time (t) is not included.



x

$u_x = u\cos\theta$

$a_x = 0$

t

$s = ut + \frac{1}{2}at^2$

$x = u\cos\theta\, t$

$t = \frac{x}{u\cos\theta}$

y

$u_y = u\sin\theta$

$a_y = -g$

t

$s = ut + \frac{1}{2}at^2$

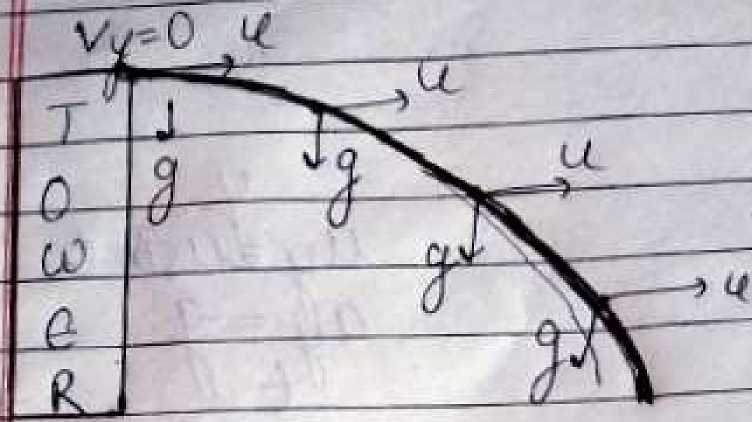
$y = u\sin\theta\, t + \frac{1}{2}(-g)t^2$

$$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\cdot\frac{x^2}{u^2\cos^2\theta}$$

$$\boxed{y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}}$$ (Parabola)

This is called Equation of Trajectory.

Projectile from Height.

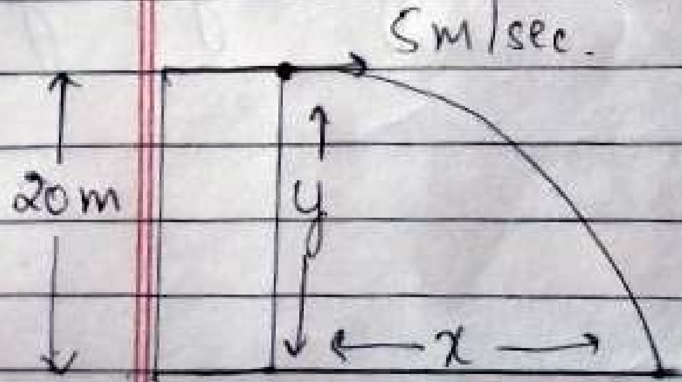


2-D motion

Velocity in x-direction remains constant whereas velocity in y direction increases (due to g)

Path → Parabolic path (projectile from height)

Ques- consider a building of 20m, from the top of building we threw a particle horizontally at the speed of 5m/sec. Find the time taken by particle to reach ground.



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Ans8- It is a 2-D motion (covers a distance in x & y direction).

Let us consider whole motion in y-direction.

$S_y = -20m$ (displacement)

$u_y = 0$, $a_y = -g = -10$

$s = ut + \frac{1}{2}at^2$

$-20 = 0 + \frac{1}{2}(-10)t^2$

$\boxed{t = 2 sec}$

⇒ Find the Horizontal range.

Let us consider motion in x-direction

$S_x = R$
$u_x = 5 m/sec$
$a_x = 0$, $t = 2sec$

$s = ut + \frac{1}{2}at^2$

$R = 5 \times 2 + 0$
$\boxed{R = 10m}$

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⇒ Find the velocity of ball when it hits the ground

$\vec{v} = \vec{v}_x + \vec{v}_y$

Y-direction

$u_y = 0$

$a_y = -10$

$v_y = ?$

$v = u + at$

$v = -20 \, m/sec$

$\vec{v} = \vec{v}_x + \vec{v}_y$

$$\boxed{\vec{v} = 5\hat{i} - 20\hat{j}}$$

⇒ Speed

Speed = Mag. of velocity

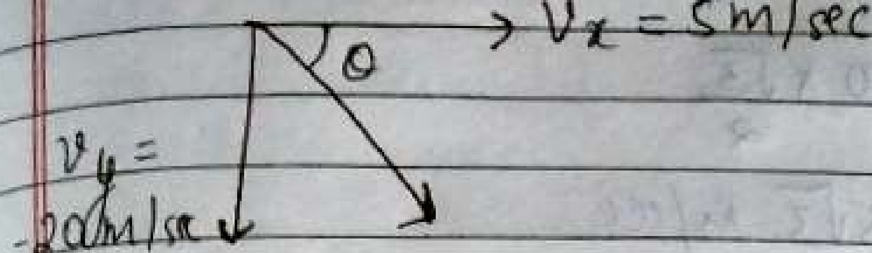
Speed = $\sqrt{(5)^2 + (-20)^2}$

Speed = $\sqrt{25 + 400}$

Speed = $\sqrt{425}$

"Whenever you take a step forward, you are bound to disturb something."

→ Find θ.



$$\tan\theta = \frac{v_y}{v_x}$$

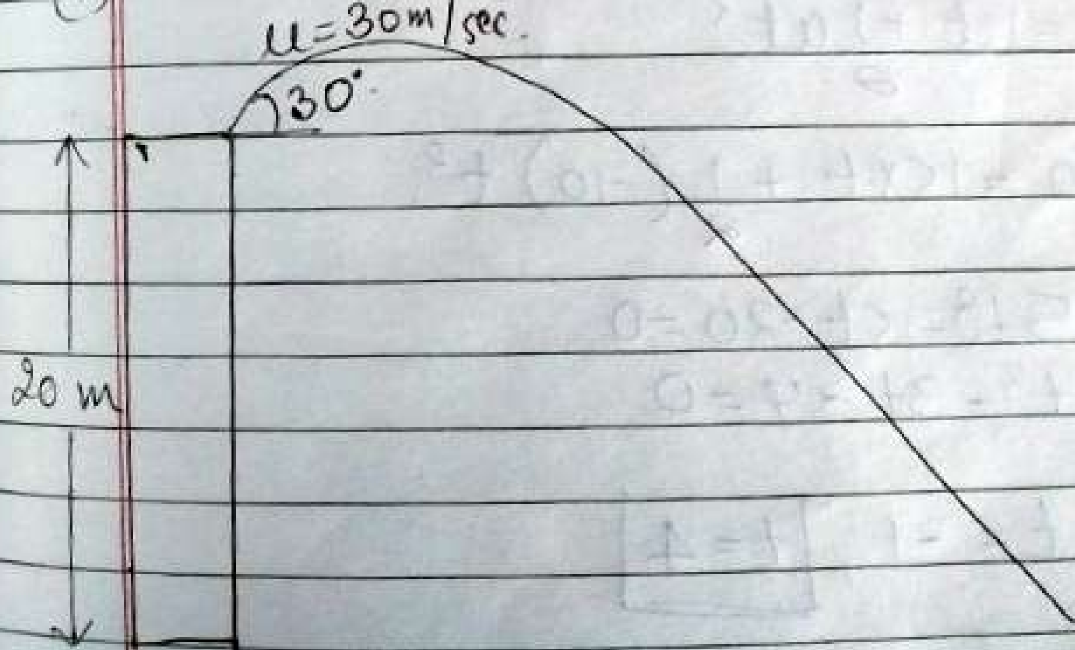
$$\tan\theta = \frac{20}{5}$$

$$\tan\theta = 4$$

$$\theta = \tan^{-1}4$$

Ques 8- If a ball is thrown at 30° angle from a (20m) tower with initial speed 30m/sec.

(i) Find the time taken to reach ground.



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In x-direction

$u_x = u\cos 30^\circ$

$u_x = 30 \times \frac{\sqrt{3}}{2}$

$u_x = 15\sqrt{3}$ m/sec.

In y-direction

$u_y = u\sin\theta$

$u_y = 15$ m/sec.

(i) Let us take motion in y-direction

$S_y = -20$ (shortest between initial and final position)

$a_y = -10$

$S = ut + \frac{1}{2}at^2$

$-20 = 15 \times t + \frac{1}{2}(-10)t^2$

$5t^2 - 15t - 20 = 0$

$t^2 - 3t - 4 = 0$

$t = -1, \boxed{t = 4}$

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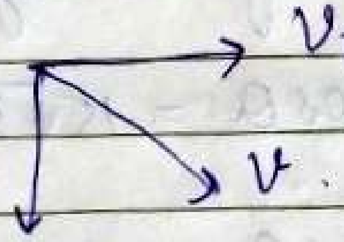
(ii) find Horizontal range

$u_x = 15\sqrt{3}$.
$a_x = 0$
$t = 4$ sec.
$S_x = R$

$$S = ut + \frac{1}{2}at^2$$

$$\boxed{R = 60\sqrt{3}\ m}$$

(iii) final velocity just before hitting ground.



Y-direction
$u_y = 15$
$a_y = -10$
$t = 4$ sec.
$V_y = ?$, $V = u + at$
$V = -25$ m/sec.

$$\boxed{V_y = -25\ m/sec}$$

Velocity $= 15\sqrt{3}\hat{i} - 25\hat{j}$.

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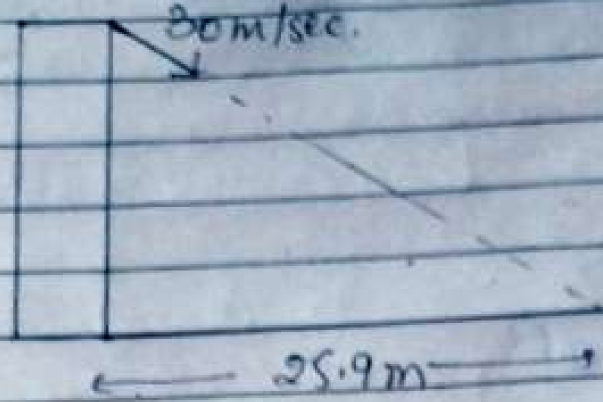
Ques:-



Find the height of tower?

Ans:- Component of velocity.

$u_x = u\cos\theta = 15\sqrt{3}$ m/sec

$u_y = u\sin\theta = 15$ m/sec.

X-direction motion

$u_x = 15\sqrt{3}$

$a = 0$

$S_x = 25.9$

$t = ?$

$s = ut + \frac{1}{2}at^2$

$t = \frac{25.9}{15\sqrt{3}}$

Y-direction motion

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$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{15 \times 2 \cdot 59}{15\sqrt{3}} + \frac{1}{2}(-10) \times t^2$$

EQUATION OF TRAJECTORY.

$$\left\{ y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \right\}$$

$$y = x\tan\theta - \frac{gx^2 \sin\theta}{2u^2\cos^2\theta \sin\theta}$$

$$y = x\tan\theta - \frac{gx^2\tan\theta}{u^2 2\sin\theta\cos\theta}$$

$$y = x\tan\theta - \frac{x^2\tan\theta}{\frac{u^2\sin 2\theta}{g}}$$

$$y = x\tan\theta - \frac{x^2\tan\theta}{R}$$

$$\left\{ y = x\tan\theta\left(1 - \frac{x}{R}\right) \right\}$$

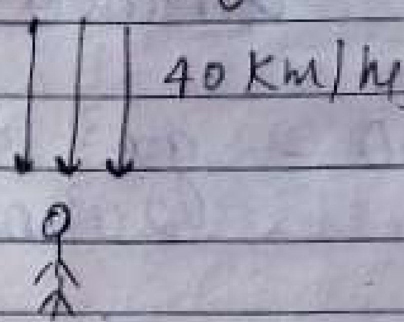
This is also equation of trajectory.

"No religion has mandated killing others as a requirement for its sustenance or promotion." —Dr.A.P.J.Abdul Kalam

Relative Velocity 2-D, Rain Man Problem, Umbrella Woman Problem.

$\boxed{V_{AB} = V_{AG} - V_{BG}}$ 2-D

Ques- The speed of rain is 40 km/hr. A man is standing on ground. What is the speed of Rain as seen by man



Ans- $V_{Rm} = V_{RG} - V_{mG}$

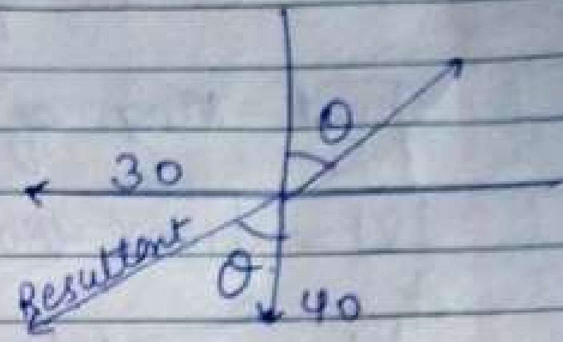
$V_{Rm} = -40\hat{j} - 0$

$V_{Rm} = -40\hat{j}$ (The man will open its umbrella opp. to the direction of rain).

⇒ Now the man is in a cycle with speed 30 km/hr at what angle should he bend his umbrella from vertical.

$v_{Rm} = v_{RG} - v_{mG}$

$v_{Rm} = -40\hat{j} - 30\hat{i}$

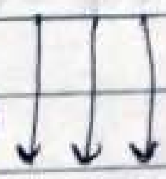


$\tan\theta = \frac{30}{40} = \frac{3}{4}$

$\theta = 37^\circ$.

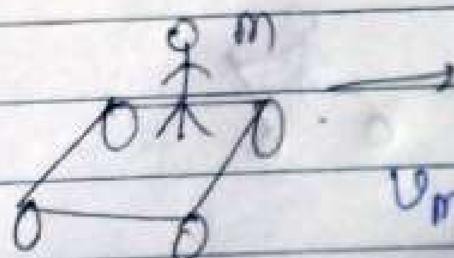
The man should hold the umberella 30° from vertical position

Ques 6 - If the speed of rain is 60 km/hr vertically. A man is in trolley of speed 80 km/hr, At what angle will the man open's its umberella.



$R = 60$ km/hr.

$v_{RG} = -60\hat{j}$

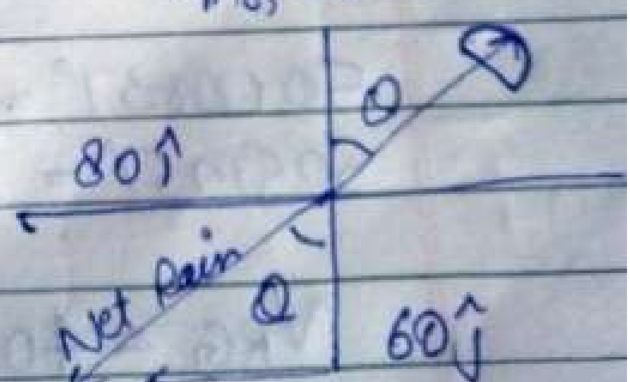


80 km/hr.

$v_{mG} = 80\hat{i}$

$v_{Rm} = v_{RG} - v_{mG}$

$v_{Rm} = -60\hat{j} - 80\hat{i}$.



$\tan\theta = \frac{80}{60} = \frac{4}{3}$.

$\theta = 53^\circ$.

⇒ what is the speed of rain w.r.t man

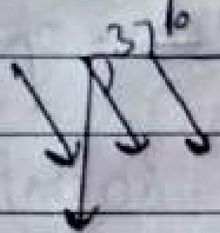
speed = Mag. of velocity

$|v| = \sqrt{(60)^2 + (80)^2}$

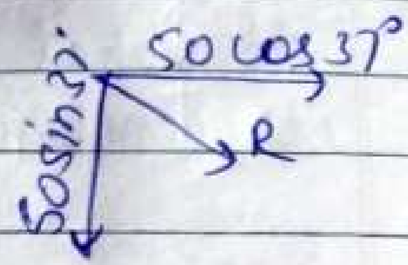
$|v| = 100$ ~~km/hr~~

$|v| = 100 \text{ km/hr}$

Ques 6 - If rain is at 37° angle & $V_{RG} = 50$ km/hr a man in cycle at a speed of 40 km/hr V_{mG} (+x direction). find the position of umberalla.



$V_{RG} = 50$ km/hr



$50\cos 37^\circ = 40$

$50\sin 37^\circ = 30$

$V_{RG} = 40\hat{i} - 30\hat{j}$

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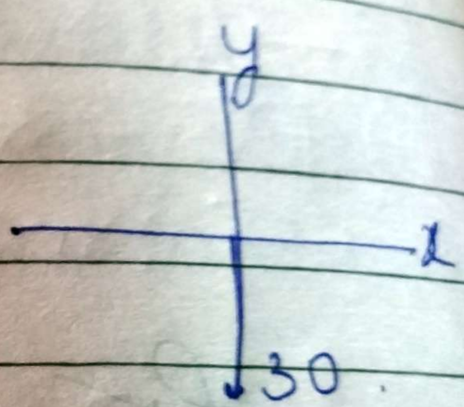
$V_{Rm} = V_{RG} - V_{mG}$

$= 40\hat{i} - 30\hat{j} - 40\hat{i}$

$V_{Rm} = -30\hat{j}$



Umberalla should be verticALLy upward.